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Pythagorean fuzzy set, Pythagorean fuzzy topology, Maximal open set, Minimal open set, Fuzzy topological space, Generalized topology, uncertainty modelling.

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Pythagorean Fuzzy Nano Topological Space With (Min-Max Open Set)

Abstract

Pythagorean fuzzy set, characterized by the condition that the sum of the squares of membership and non-membership degrees does not exceed one, provide a more flexible framework for modeling uncertainty than intuitionistic fuzzy sets. In this paper, we introduce and investigate the concept of maximal open sets in Pythagorean fuzzy topological spaces. Several properties and fundamental are established, and their relationships with Pythagorean fuzzy open and closed sets are examined. Characterizations of maximal and minimal open sets are obtained through inclusion relations and intersection properties. Furthermore, illustrative examples are provided to demonstrate the applicability of the proposed concepts. The developed theory extends classical topological notions to the Pythagorean fuzzy setting and contributes to the study of generalized topological structures under uncertainty. Potential applications of the proposed framework in decision-making, information systems, and intelligent data analysis are also discussed, offering new directions for future research in fuzzy topology and soft computing.

1. INTRODUCTION

The concept of fuzzy minimal and maximal open sets introduced by Ittanagi and Wali in [] has played an important role. Swaminathan and Sivaraja [] discussed some properties of fuzzy minimal open and fuzzy maximal open sets. In this chapter, we have introduced the notions of Pythagorean fuzzy nano minimal (resp., δ , δ -pre, δ -semi, $\delta\alpha$, and $\delta\beta$)-open sets. Further, we have developed Pythagorean fuzzy nano maximal (resp., δ , δ -pre, δ -semi, $\delta\alpha$, and $\delta\beta$)-open sets and various interesting results on them.

2 .PYTHAGOREAN FUZZY NANO MINIMAL OPEN SETS

Definition 2.2.1 : Let $(U, \tau_R A)$ be a $P\mathcal{N}$ ts with respect to A where A is a $P\mathcal{S}$ of U . Then $P\mathcal{S}$ S in U is said to be Pythagorean:

- (i) fuzzy nano $\delta\alpha$ or α -open set (briefly, $P\mathcal{N}\delta\alpha$ os or $P\mathcal{N}\alpha$ os) if $S \subseteq P\mathcal{N}int(P\mathcal{N}cl(P\mathcal{N}int(S)))$.
- (ii) fuzzy nano $\delta\beta$ ore*-open set (briefly, $P\mathcal{N}\delta\beta$ os or $P\mathcal{N}e^*os$) if $S \subseteq P\mathcal{N}cl(P\mathcal{N}int(P\mathcal{N}cl(S)))$.

The complement of $P\mathcal{N}\delta\alpha$ cs, $P\mathcal{N}\delta\beta$ os and $P\mathcal{N}\delta\beta$ cs) of U is denoted by $P\mathcal{N}\delta\alpha OSU$ (res. $P\mathcal{N}\delta\alpha CSU$, $P\mathcal{N}\delta\beta OSU$ and $P\mathcal{N}\delta\beta CSU$).

Definition 2.2.2: Let $(U, \tau_R A)$ be a $P\mathcal{N}$ ts with respect to A where A is a $P\mathcal{S}$ of U . Let S be a $P\mathcal{S}$ of U . Then Pythagorean fuzzy nano

- (i) $\delta\alpha$ (resp. $\delta\beta$) interior of S (briefly, $P\mathcal{N}\delta\alpha int S$ (res. $P\mathcal{N}\delta\beta int(S)$)) is defined by $P\mathcal{N}\delta\alpha int(S)$ (resp. $P\mathcal{N}\delta\beta int(S) = \bigcup \{I : I \subseteq S \text{ \& } I \text{ is a } P\mathcal{N}\delta\alpha \text{ (resp. } P\mathcal{N}\delta\beta) \text{ set in } U\}$.
- (ii) $\delta\alpha$ (resp. $\delta\beta$) closure of S (briefly, $P\mathcal{N}\delta\alpha cl S$ (res. $P\mathcal{N}\delta\beta cl(S)$)) is defined by $P\mathcal{N}\delta\alpha cl(S)$ (resp. $P\mathcal{N}\delta\beta cl(S) = \bigcup \{A : S \subseteq A \text{ \& } A \text{ is a } P\mathcal{N}\delta\alpha \text{ (resp. } P\mathcal{N}\delta\beta) \text{ set in } U\}$.

Definition 2.2.3: A proper nonzero $P\mathcal{N}\delta\beta$ os (resp. $P\mathcal{N}\delta\alpha$ os, $P\mathcal{N}\delta\beta$ os, $P\mathcal{N}\delta\alpha$ os and $P\mathcal{N}\delta\beta$ os) E in Pythagorean fuzzy nano topological space $(U, \tau_R A)$ is said to be Pythagorean fuzzy nano minimal (resp., δ , δ -pre, δ -semi, $\delta\alpha$, and $\delta\beta$)-open sets (briefly, $P\mathcal{N}minos$ (resp. $P\mathcal{N}\delta min\delta os$, $P\mathcal{N}min\delta Pos$, $P\mathcal{N}min\delta Sos$, $P\mathcal{N}min\delta\alpha os$ and $P\mathcal{N}min\delta\beta os$) if a $P\mathcal{N}os$ (resp. $P\mathcal{N}\delta os$, $P\mathcal{N}\delta Pos$, $P\mathcal{N}\delta Sos$ and $P\mathcal{N}\delta\beta os$) contained in E is $0P$ or E .

Example 2.2.4: Let $U = \{s_1, s_2, s_3, s_4\}$ be the universe set and the equivalence relation $U/R = s_1, s_4, s_2, s_3$. Let $A = s_{10.8, 0.4}, s_{20.6, 0.6}, s_{30.7, 0.7}, s_{40.5, 0.7}$ be a Pythagorean fuzzy subsets of U .

$$P\mathcal{N}A = s_{10.5, 0.7}, s_{20.6, 0.6}, s_{30.7, 0.7},$$

$$P\mathcal{N}A = s_{10.8, 0.4}, s_{20.6, 0.6}, s_{30.7, 0.7},$$

$$\mathcal{B}P\mathcal{N}(A) = s_{10.7, 0.5}, s_{20.6, 0.6}, s_{30.7, 0.7}.$$

$$\text{Now } \tau_R A = \{0P, 1P, P\mathcal{N}A_1, P\mathcal{N}A_1, \mathcal{B}P\mathcal{N}A_1\}$$

Here (i) $P\mathcal{N}A$ is $P\mathcal{N}min\delta o$, $P\mathcal{N}\delta So$ and $P\mathcal{N}\delta\alpha o$ set, (ii) $(P\mathcal{N}(A))^c$ is a $P\mathcal{N}min\delta Po$ and $P\mathcal{N}min\delta\beta o$ set.

Lemma 2.2.5: Let $(U, \tau_R A)$ be a Pythagorean fuzzy nano topological space.

- (i) If E_1 is $P\mathcal{N}minos$ (resp. $P\mathcal{N}minos$, $P\mathcal{N}min\delta Pos$, $P\mathcal{N}min\delta Sos$, $P\mathcal{N}min\delta\alpha os$ and $P\mathcal{N}min\delta\beta os$) and E_2 is $P\mathcal{N}os$ (resp. $P\mathcal{N}os$,

$P\mathcal{N}\delta os, P\mathcal{N}\delta Sos, P\mathcal{N}\delta\alpha os$) in $U, \tau_R A$, then $E_1 \cap E_2 = 0P$ or $E_1 \subseteq E_2$.

(ii) If E_1 and E_2 are $P\mathcal{N}minos$ (resp. $P\mathcal{N}min\delta os, P\mathcal{N}\delta min\delta Pos, P\mathcal{N}min\delta Sos, P\mathcal{N}min\delta\alpha os$ and $P\mathcal{N}min\delta\beta os$), then $E_1 \cap E_2 = 0P$ or $E_1 = E_2$.

Proof:

(i) Let us assume that E_2 is $P\mathcal{N}\delta\beta os$ in $U, \tau_R A$ such that $E_1 \cap E_2 \neq 0P$. Since E_1 is $P\mathcal{N}min\delta\beta os$, and $E_1 \cap E_2 \subseteq E_1$, then $E_1 \cap E_2 = E_1$ implies $E_1 \subseteq E_2$.

(ii) Suppose that $E_1 \subseteq E_2 \neq 0P$, then clearly from (i), $E_1 \subseteq E_2$ and $E_2 \subseteq E_1$ as E_1 and E_2 are

$P\mathcal{N}min\delta\beta os$. Hence $E_1 = E_2$. The proof of the others are similar.

Theorem 2.2.6: Let E and E_i are $P\mathcal{N}minos$ (resp. $P\mathcal{N}min\delta os, P\mathcal{N}\delta min\delta Pos, P\mathcal{N}min\delta Sos, P\mathcal{N}min\delta\alpha os$ and $P\mathcal{N}min\delta\beta os$) for any $i \in M$. If $E \subseteq i \in M E_i$, then $E = E_j$ for any $j \in M$.

Proof: Suppose $E \subseteq i \in M E_i$, then $E = E \cap (i \in M E_i) = i \in M (E \cap E_i)$. By Lemma 2.2.5(ii), $E \cap E_i = 0P$ or $E = E_i$ as E and E_i are $P\mathcal{N}min\delta\beta os$'s. If $E \cap E_i = 0P$, then $E = 0P$ which contradicts that E is a $P\mathcal{N}min\delta\beta os$. Hence if $E \cap E_i \neq 0P$ then $E = E_j$ for any $j \in M$. The proof of the others are similar.

Theorem 2.2.7: If E and E_i are $P\mathcal{N}minos$ (resp. $P\mathcal{N}min\delta os, P\mathcal{N}\delta min\delta Pos, P\mathcal{N}min\delta Sos, P\mathcal{N}min\delta\alpha os$ and $P\mathcal{N}min\delta\beta os$) for any $i \in M$ and $E \neq E_i$, then $E \cap (i \in M E_i) = 0P$ for any $i \in M$.

Proof: Let $E \cap (i \in M E_i) \neq 0P$, then $E \cap E_i \neq 0P$ for any $i \in M$. By Lemma 2.2.5(ii), $E = E_i$ contradictory to $E \neq E_i$. Hence $E = E \cap (i \in M E_i) = 0P$.

Theorem 2.2.8: If E_i is a $P\mathcal{N}minos$ $P\mathcal{N}\delta min\delta Pos, P\mathcal{N}min\delta Sos, P\mathcal{N}min\delta\alpha os$ and $P\mathcal{N}min\delta\beta os$) for any $i \in M (M \geq 2)$ and $E_i \neq E_j$ for any distinct $i, j \in M$, then $((i \in M / \{j\}) E_i) \cap E_j = 0P$ for any $j \in M$.

Proof: Let $((i \in M / \{j\}) E_i) \cap E_j \neq 0P$. Then $i \in M / \{j\} (E_i \cap E_j) \neq 0P \Rightarrow E_i \cap E_j \neq 0P$. By Lemma 2.2.5 (ii), $E_i = E_j$, a contradiction. Hence $((i \in M / \{j\}) E_i) \cap E_j = 0P$ for any $j \in M$.

Theorem 2.2.9: If E_i is a $P\mathcal{N}minos$ (resp. $P\mathcal{N}\delta min\delta Pos, P\mathcal{N}min\delta Sos, P\mathcal{N}min\delta\alpha os$ and $P\mathcal{N}min\delta\beta os$) for any $i \in M (M \geq 2)$ and $E_i \neq E_j$ for any distinct $i, j \in M$. If K is a nonempty proper subset of M then $((i \in M / K) E_i) \cap E_j = 0P$.

Proof: If $((i \in M / K) E_i) \cap E_j \neq 0P$. It implies that $(E_i \cap E_j) \neq 0P$ for $i \in M / K$ and $s \in K$ implies that $E_i \cap E_s \neq 0P$ for some $i \in M$ and $j \in K$. By Lemma 2.2.5 (ii), $E_i = E_j$, which is a contradiction. Hence $((i \in M / K) E_i) \cap E_j = 0P$.

Theorem 2.2.10: If E_i and E_k are $P\mathcal{N}minos$ (resp. $P\mathcal{N}\delta min\delta Pos, P\mathcal{N}min\delta Sos, P\mathcal{N}min\delta\alpha os$ and $P\mathcal{N}min\delta\beta os$) for any $i \in M$ and $k \in S$ and if there exists an $n \in S$ such that $E_i \neq E_j$ for any $i \in M$, then $n \in K \cap S \subseteq [i \in M E_i]$.

Proof: Assume that there exists an $n \in S$ such that $E_i \neq E_n$ for any $i \in M$, then $n \in K \cap S \not\subseteq [i \in M E_i]$. $\Rightarrow E_n \subseteq [i \in M E_i]$ for some $n \in K$.

$\Rightarrow E_i \neq E_n$ for any $i \in M$, by Theorem 2.2.6, which is a contradiction. Hence $n \in K \cap \{i \in M \mid E_i\}$.

Theorem 2.2.11: If E_i is a $P\mathcal{N}$ minos (resp. $P\mathcal{N}$ minδos, $P\mathcal{N}$ δminδPos, $P\mathcal{N}$ minδSos, $P\mathcal{N}$ minδaos and $P\mathcal{N}$ minδβos) for any $i \in M$ such that $E_i \neq E_j$ for any distinct $i, j \in M$, then

(i) $E_j \subset [i \in M / \{j\} E_i]$ for some $j \in M$.

(ii) $i \in M / \{j\} E_i \neq 1P$ for any $j \in M$.

Proof:

(i) By hypothesis, $E_i \neq E_j$ for any distinct $i, j \in M$. By Theorem 2.2.7, $i \in M \cap E_j = 0P$ which is true for any $j \in M$. $\Rightarrow i \in [E_i \cap E_j] = 0P$

$\Rightarrow E_i \cap E_j = 0P$ (By Lemma 2.2.5(ii))

$\Rightarrow E_i \subset E_j^c$

$\Rightarrow i \in M / \{j\} E_i \subset E_j^c$. Hence proved.

(ii) Let $j \in M$ such that $i \in M / \{j\} = 1P$

$\Rightarrow E_i = 0P$

$\Rightarrow E_i$ is not a $P\mathcal{N}$ minδβos set, a contradiction. Hence $i \in M / \{j\} \neq 1P$ for any $j \in M$. The proof of others are similar.

Corollary 2.2.12: If E_i is a $P\mathcal{N}$ minos (resp. $P\mathcal{N}$ minδos, $P\mathcal{N}$ δminδPos, $P\mathcal{N}$ minδSos, $P\mathcal{N}$ minδaos and $P\mathcal{N}$ minδβos) for any $i \in M$ such that $E_i \neq E_j$ for any distinct $i, j \in M$, then $E_i \cup E_j \neq 1P$ for any distinct $i, j \in M$.

Proof: Similar to the proof of Theorem 2.2.12 (ii).

Theorem 2.2.13: If E_i is a $P\mathcal{N}$ minos (resp. $P\mathcal{N}$ minδos, $P\mathcal{N}$ δminδPos, $P\mathcal{N}$ minδSos, $P\mathcal{N}$ minδaos and $P\mathcal{N}$ minδβos) for any $i \in M$ such that $E_i \neq E_j$ for any distinct $i, j \in M$, then $E_j = [i \in M \cap E_i] \cap [i \in M / \{j\} E_i]^c$ for any $j \in M$.

Proof: For any $j \in M \Rightarrow [i \in M \cap E_i] \cap [i \in M / \{j\} E_i]^c = [i \in M / \{j\} E_i \cup E_j] \cap [i \in M / \{j\} E_i \cup E_j]^c = [(i \in M \cap E_i) \cap (i \in M / \{j\} E_i)^c] \cup [E_j \cap (E_j \cap (i \in M / \{j\} E_i)^c)] = 0P \cup E_j = E_j$ for any $j \in M$.

Proposition 2.2.14: Let G be a $P\mathcal{N}$ minos (resp. $P\mathcal{N}$ minδos, $P\mathcal{N}$ δminδPos, $P\mathcal{N}$ minδSos, $P\mathcal{N}$ minδaos and $P\mathcal{N}$ minδβos) set in $U, \tau PA$. If $x\alpha \in G$, then $G \subset G_1$ for any $P\mathcal{N}$ -open neighbourhood of $x\alpha$ for any $x\alpha \in G$.

Proof: Let G_1 be an $P\mathcal{N}$ -open neighbourhood of $x\alpha$ such that $G \not\subset G_1 \subset G$ and $G \cap G_1 \neq 0P$. This implies that G is a $P\mathcal{N}$ minδβos set which is contradiction. The proof of others are similar.

Proposition 2.2.15: Let G be a $P\mathcal{N}$ minδβos (resp. $P\mathcal{N}$ minδos, $P\mathcal{N}$ δminδPos, $P\mathcal{N}$ minδSos, $P\mathcal{N}$ minδaos and $P\mathcal{N}$ minδβos). Then $G = \bigcap \{G_1 : G_1 P\mathcal{N}\delta (\text{ resp. } P\mathcal{N}\delta P, P\mathcal{N}\delta S, P\mathcal{N}\delta\alpha \text{ and } P\mathcal{N}\delta\beta)\text{-open neighbourhood of } x\alpha \text{ for any } x\alpha \in G\}$.

Proof: By Proposition 2.2.14 and as G is an $P\mathcal{N}$ δβ-open neighbourhood of $x\alpha$, we have

$G = \bigcap \{G_1 : G_1 P\mathcal{N}\delta\beta\text{-open neighbourhood of } x\alpha \subset G\}$. This completes the proof. The proof of others are similar.

Theorem 2.2.16: Let G be a $P\mathcal{N}$ minos (resp. $P\mathcal{N}$ minδos, $P\mathcal{N}$ δminδPos, $P\mathcal{N}$ minδSos,

$P\mathcal{N}$ minδaos and $P\mathcal{N}$ minδβos). Then the following conditions are equivalent.

(i) G is a $P\mathcal{N}$ minos (resp. $P\mathcal{N}$ minδos, $P\mathcal{N}$ δminδPos, $P\mathcal{N}$ minδSos, $P\mathcal{N}$ minδaos and $P\mathcal{N}$ minδβos) set.

(ii) $G \subset P\mathcal{N}Cl(K)$ (resp. $G \subset P\mathcal{N}\delta ClK, \subset P\mathcal{N}\delta PCIK \subset P\mathcal{N}\delta SCIK, \subset P\mathcal{N}\delta\alpha ClK$ and $\subset P\mathcal{N}\delta\beta Cl(K)$ for any nonzero $P\mathcal{S}$ of K of G .

(iii) $P\mathcal{N}ClG = P\mathcal{N}ClK$ (resp. $P\mathcal{N}\delta ClG = P\mathcal{N}\delta ClK, P\mathcal{N}\delta PCIG = P\mathcal{N}\delta PCIK, P\mathcal{N}\delta SCIG = P\mathcal{N}\delta SCIK, P\mathcal{N}\delta\alpha ClG = P\mathcal{N}\delta\alpha ClK$ and

$P\mathcal{N}\delta\beta ClG = P\mathcal{N}\delta\beta Cl(K)$ for any nonzero $P\mathcal{S} \subseteq G$.

Proof: (i) $\Rightarrow$ (ii): By Proposition 2.2.14 for any $x\alpha \in G$ and $P\mathcal{N}\delta\beta$ open neighbourhood M of $x\alpha$, we have $K = G \cap K \subset (M \cap K)$ for any proper nonzero $P\mathcal{N}K \subset G$. Therefore, we have $(M \cap K) \neq 0P$ and $x\alpha \subset P\mathcal{N}\delta\beta ClK$. It follows that $G \subset P\mathcal{N}\delta\beta ClK$.

(ii) $\Rightarrow$ (iii): For any $P\mathcal{S} \subseteq K$ of G , $P\mathcal{N}\delta\beta ClG \subset P\mathcal{N}\delta\beta ClK$. Also by (iii) $P\mathcal{N}\delta\beta ClG \subset P\mathcal{N}\delta\beta ClK = P\mathcal{N}\delta\beta ClK$. Hence proved.

(iii) $\Rightarrow$ (i): Let us assume that G is not $P\mathcal{N}$ minδβos. Then there exists a proper $P\mathcal{N}$ minδβos D such that $D \subset G$. Then there exists $y\alpha \in G$ such that $y\alpha \notin D$. Then $P\mathcal{N}\delta\beta Cl(y\alpha) \in D^c$ implies that $P\mathcal{N}\delta\beta Cl y\alpha \neq P\mathcal{N}\delta\beta ClG$, a contradiction. This completes proof. The proof of others are similar.

2.3 PYTHAGOREAN FUZZY NANO MAXIMAL OPEN SETS AND ITS PROPERTIES

Definition 2.3.1: A proper nonzero Pythagorean fuzzy nano (resp. δ , δpre , δ semi, $\delta\alpha$ and $\delta\beta$)- open set (briefly, $P\mathcal{N}$ nos (resp. $P\mathcal{N}$ nos, $P\mathcal{N}\delta Pos$, $P\mathcal{N}\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}\delta\beta os$) F of a Pythagorean fuzzy nano topological space $U, \tau PA$ is said to Pythagorean fuzzy nano Maximal (resp. $P\mathcal{N}$ maxδos, $P\mathcal{N}$ maxδPos, $P\mathcal{N}$ maxδSos, $P\mathcal{N}$ maxδaos and $P\mathcal{N}$ maxδβos)) if any $P\mathcal{N}$ nos (resp. $P\mathcal{N}\delta os$, $P\mathcal{N}\delta Pos$, $P\mathcal{N}\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}\delta\beta os$) which contains F is either F or $1P$.

Example 2.3.2: In Example 2.2.4, the set (i) $BP\mathcal{N}(A)$ is $P\mathcal{N}$ maxδo and $P\mathcal{N}$ maxδSo, (ii) $P\mathcal{N}(A)$ is $P\mathcal{N}$ maxδPo, $P\mathcal{N}$ maxδao and $P\mathcal{N}$ maxδβo set.

Lemma 2.3.3 : Let $U, \tau PA$ be a Pythagorean fuzzy nano topological space. Then

(i) If F_1 is a $P\mathcal{N}$ maxos (resp. $P\mathcal{N}$ maxδos, $P\mathcal{N}$ maxδPos, $P\mathcal{N}$ maxδSos, $P\mathcal{N}\delta os$, $P\mathcal{N}\delta Pos$ and $P\mathcal{N}\delta\beta os$) in U , then $F_1 \cup F_2 = 1P$ or $F_2 \subset F_1$.

(ii) If F_1 and F_3 are $P\mathcal{N}$ maxos (resp. $P\mathcal{N}$ maxδos, $P\mathcal{N}$ maxδPos, $P\mathcal{N}$ maxδSos, $P\mathcal{N}\delta os$, $P\mathcal{N}\delta Pos$ and $P\mathcal{N}\delta\beta os$) then either $F_1 \cup F_3 = 1P$ or $F_1 = F_3$.

Proof: (i) Assume that $F_2 \not\subset F_1$. Clearly, $F_1 \subset F_1 \cup F_2$ a contrary to F_1 is a $P\mathcal{N}$ maxδβos if $F_1 \cup F_2 \neq 1P$. Hence, $F_1 \cup F_2 = 1P$.

(ii) Let F_1 and F_3 are $P\mathcal{N}$ maxδβos's. Then from (i) $F_3 \subset F_1$ and $F_1 \subset F_3$ implies that $F_1 = F_3$. The proof of others are similar.

Theorem 2.3.4: If F_1, F_2 and F_3 are $P\mathcal{N}maxos$ (resp. $P\mathcal{N}max\delta os$, $P\mathcal{N}max\delta Pos$, $P\mathcal{N}max\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}max\delta\beta os$) such that $F_1 \neq F_2$ and $(F_1 \cap F_2) \subset F_3$, then their $F_1 = F_3$ or $F_2 = F_3$.

Proof: Suppose that F_1, F_2 and F_3 are $P\mathcal{N}max\delta\beta os$'s with $F_1 \neq F_2, F_1 \cap F_2 \subset F_3$, and if $F_1 \neq F_3$, then

$$\begin{aligned} F_2 \cap F_3 &= F_2 \cap F_3 \cap 1P \\ &= F_2 \cap [F_3 \cap (F_1 \cup F_2)], \text{ by Lemma 2.3.3 (ii)} \\ &= F_2 \cap F_3 \cap F_1 \cup (F_3 \cap F_2) \\ &= [F_2 \cap F_3 \cap F_1] \cup [F_2 \cap F_3 \cap F_2] \\ &= [(F_2 \cap F_1)] \cup [F_2 \cap F_3] \\ &= F_2 \cap [F_1 \cup F_3] \\ &= F_2 \cap 1P \\ &= F_2 \end{aligned}$$

$F_2 \cap F_3 = F_2 \Rightarrow F_2 \subset F_3$. As F_2 and F_3 are $P\mathcal{N}max\delta\beta os$'s, $F_3 \subset F_2$. Hence $F_2 = F_3$.

Theorem 2.3.5: For any distinct $P\mathcal{N}maxos$ (resp. $P\mathcal{N}max\delta os$, $P\mathcal{N}max\delta Pos$, $P\mathcal{N}max\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}max\delta\beta os$) such that F_1, F_2 and F_3 we have $F_1 \cap F_2 \not\subset F_1 \cap F_3$.

Proof: Consider $F_1 \cap F_2 \subset F_1 \cap F_3$ for any distinct $P\mathcal{N}max\delta\beta os$'s F_1, F_2 and F_3 . Then $F_1 \cap F_2 \cup F_2 \cap F_3 \subset F_1 \cap F_3 \cup F_2 \cap F_3 = F_1 \cap F_3 \cap F_2 \subset F_1 \cup F_2 \cap F_3 = 1P \cap F_2 \subset 1P \cap F_3 = F_2$ is contained in F_3 a contradiction to F_1, F_2 and F_3 are distinct. Hence $F_1 \cap F_2 \neq F_1 \cap F_3$.

Theorem 2.3.7: If F_i is a $P\mathcal{N}maxos$ (resp. $P\mathcal{N}max\delta os$, $P\mathcal{N}max\delta Pos$, $P\mathcal{N}max\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}max\delta\beta os$)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, then

- (i) $[i \in M \mid F_i]_c \subset F_j$ for any $j \in M$.
- (ii) $i \in M \setminus \{j\} \mid F_i \neq 0P$ for any $j \in M$.

Corollary 2.3.8: If F_i is a $P\mathcal{N}maxos$ (resp. $P\mathcal{N}max\delta os$, $P\mathcal{N}max\delta Pos$, $P\mathcal{N}max\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}max\delta\beta os$)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, then $F_i \cap F_j \neq 0P$ for any distinct $i, j \in M$.

Remark 2.3.9 : If F_i is a $P\mathcal{N}maxos$ (resp. $P\mathcal{N}max\delta os$, $P\mathcal{N}max\delta Pos$, $P\mathcal{N}max\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}max\delta\beta os$)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, and if K is a proper nonzero fuzzy subset of M , then $i \in M \setminus K \mid F_i \notin K$.

Theorem 2.3.10: If F_i is a $P\mathcal{N}maxos$ (resp. $P\mathcal{N}max\delta os$, $P\mathcal{N}max\delta Pos$, $P\mathcal{N}max\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}max\delta\beta os$)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, and if $i \in M \setminus F_i$ is a fuzzy subset, then F_j is a fuzzy subset for any $j \in M$.

Proof: By theorem 2.3.9, we have $F_j = i \in M \setminus F_i \cup [i \in M \setminus \{j\} \mid F_i]_c$ for any $j \in M$, $F_j = [i \in M \setminus F_i \cup [i \in M \setminus \{j\} \mid F_i]_c]$.

Since M is finite $i \in M \setminus \{j\} \mid F_i$ is a $P\mathcal{N}max\delta\beta$ -closed. Hence F_j is $P\mathcal{N}max\delta\beta$ -closed for any $j \in M$.

Similarly proof the remaining theorem.

Theorem 2.3.11: If F_i is a $P\mathcal{N}maxos$ (resp. $P\mathcal{N}max\delta os$, $P\mathcal{N}max\delta Pos$, $P\mathcal{N}max\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}max\delta\beta os$)'s for any $i \in M$, M is a finite set and $F_i \neq F_j$ for any distinct $i, j \in M$, and if $i \in M \setminus F_i = 0P$, then

$F_i \mid i \in M$ is the set of all $P\mathcal{N}max\delta os$ (resp. $P\mathcal{N}max\delta Pos$, $P\mathcal{N}max\delta Sos$, $P\mathcal{N}\delta aos$ and $P\mathcal{N}max\delta\beta os$)'s of Pythagorean fuzzy nano topological space $U, \tau PA$.

Proof: Suppose that there exists another $P\mathcal{N}max\delta\beta os$ F_k of a Pythagorean fuzzy nano topological space $U, \tau PA$ such that $F_k \neq F_i, \forall i \in M$. Clearly, $0P = i \in M \setminus F_i = i \in (M \cap K) \setminus \{k\} \mid F_i \neq 0P$, by Theorem 2.3.7(ii), a contradiction. Hence $F_i \mid i \in M$ is the family of all $P\mathcal{N}max\delta\beta os$'s of Pythagorean fuzzy nano topological.

Conclusion:

In this topic, the concept of Pythagorean fuzzy nano (resp. δ , δpre , δ semi, $\delta \alpha$ and $\delta \beta$)- minimal and Pythagorean fuzzy nano (resp. δ , δpre , δ semi, $\delta \alpha$ and $\delta \beta$)-maximal open sets are introduced and some properties of Pythagorean fuzzy nano (resp. δ , δpre , δ semi, $\delta \alpha$ and $\delta \beta$)-minimal open and Pythagorean fuzzy nano (resp. δ , δpre , δ semi, $\delta \alpha$ and $\delta \beta$)-maximal open are discussed.

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