Mathematical Analysis of Occlusal Rest Design for Cast Removable Partial Dentures

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Abstract - Aims: To establish the minimum dimensions for a non-precious cast metal occlusal rest by using mathematical analysis. An occlusal rest in a removable partial denture (RPD) provides vertical support and allows occlusal forces to be transmitted through the long axis of the abutment tooth. Methods: The stress status of an occlusal rest in a posterior tooth-bounded RPD resembles that of a cantilever beam under a uniformly distributed load. A mathematical model based on a short rectangular cantilever beam was derived by using the yield strengths of a cast cobalt-chromium alloy and a cast titanium alloy (Ti-6Al-4V), with an applied static occlusal force of 400 N. Results: For the cobalt-chromium alloy, the minimum rectangular dimensions for a 2.0 mm long occlusal rest are; when the width is 2.0 mm, the depth or thickness should be at least 1.03 mm. For the titanium alloy, the minimum dimensions are; when the width is 2.0 mm, the depth or thickness should be at least 0.85 mm. A decrease in the occlusal width will increase the bending stress, requiring a thicker rest for compensation, which may lead to dentin exposure during the rest seat preparation. Conclusion: The mathematical analysis found that the traditional spoon-shaped occlusal rest seat dimensions comply with the mechanical requirements for non-precious cast metal occlusal rests in RPDs.

KEY WORDS: Occlusal rest, removable partial denture, shear stress rest

INTRODUCTION

An occlusal rest in a removable partial denture (RPD) provides vertical support and allows occlusal forces to be transmitted through the long axis of the abutment tooth. A minimum length, width and depth or thickness is needed to guarantee normal functional performance without deformation and fracture of the rest material. These requirements may conflict with the need to conserve sound tooth substance with the avoidance of dentin exposure and caries, and the desirability of having the rest restore normal tooth topography1-7.

As previously stated8, the classical recommendation for the preparation of an occlusal rest seat for a cast RPD is a rounded triangular outline form with the floor of the rest seat resembling a saucer or spoon shape3,7,9,10. A minimum buccolingual width of 2.0 to 2.5 mm2,7, with a maximum width of 1/3 of the tooth crown10 or 1/2 of the distance between the cusp tips5,9,11, is required. The mesiodistal length recommended is 1/3 to 1/2 of the crown2,4, with an occlusal reduction of 1.0 to 1.5 mm where the rest crosses the marginal ridge of the tooth2,3,7,9,12. The recommendations are based largely on clinical experience, with little scientific evidence5,13,14.

The objective of this study was to determine the optimum dimensions for a non-precious cast metal occlusal rest by using a mathematical analysis, based on the functional occlusal force, the form of the stress, and the material used for the occlusal rest.

MATERIALS AND METHODS

For the calculation of stress within a typical cast occlusal rest, the rest is modeled as a cantilever beam of uniform rectangular cross-section that is made of a linearly elastic material, and subjected to a uniformly distributed load. The base or junction between the minor connector and the occlusal rest is at the fixed end, and the rest tip is at the free end, simulating cantilever beam mechanics. When the RPD is under stress, there are various contact points between the occlusal rest and its supporting surface, namely, the rest seat preparation in the tooth.

The beam is subjected to shearing force Q(x) and bending moment M(x), measuring the distance (x) from the free end to the fixed end (Fig 1). The formulas for shear force and bending moment are15,16:

$$Q(x) = qx (0 < x < L)$$

$$M(x) = qx^2/2 (0 < x < L)$$

where q is the load per unit length, and L represents the total length of the beam.

The maximum bite force varies widely, but is in the range of approximately 100 to 800 N for natural dentitions17, and the normal chewing force is in the range of approximately 75 to 135 N from incisors to molars18. Assuming that the average maximum posterior occlusal force on a RPD is 400 N, and the occlusal rests on each side of the tooth-bounded saddle bear half of the maximum bite force equally, then W = 200 N.
Figure 1. Diagrams of shearing force $Q$ (1a) and bending moment $M$ (1b) of a cantilever beam, of length $L$. $x$ = distance from the free end ($x = 0$) to the fixed end ($x = L$).

Figure 2. Plots of occlusal width ($b$) against depth or thickness ($h$) for a cobalt-chromium (2a) and a titanium (2b) alloy, assuming an occlusal rest length of 2.0 mm.
The shearing force and bending moment are largest at the fixed end, and are of values:

\[ Q_{\text{max}} = W \] (where \( W = qL \))
\[ M_{\text{max}} = WL / 2 \] (where \( W = qL \))

The maximum shear stress (\( \tau_{\text{max}} \)) and bending stress (\( \sigma_{\text{max}} \)) formulas are:\(^1\)

\[ \tau_{\text{max}} = 3Q_{\text{max}} / 2bh = 3W / 2bh \] .................(3)

where \( b \) is the width and \( h \) is the depth or thickness of the beam.

\[ \sigma_{\text{max}} = M_{\text{max}} y / I \] (re-arranging = \( M_{\text{max}} / (I / y) \))
\[ = 6M_{\text{max}} / bh^2 \] (substituting \( I / y \))
\[ = 3WL / bh^2 \] (substituting \( M_{\text{max}} \)) ...............(4)

The maximum shear stress and the maximum normal stress due to bending of the beam must be below the design maximum limit, and these conditions govern the selection of the beam dimension. Re-arranging (3) and (4) gives:

\[ bh < 3W / 2 \tau_{\text{max}} \] ............................................(5)

\[ bh^2 < 3WL / \sigma_{\text{max}} \] ............................................(6)

Suppose we want to determine \( h \) when given \( b \), we further arrange the formulas to:

\[ h < 3W / 2b \tau_{\text{max}} \] ............................................(7)

\[ h < \sqrt(3WL / b \sigma_{\text{max}}) \] ............................................(8)

Since both conditions in (7) and (8) have to be satisfied, we arrive at the condition:

\[ h < \text{maximum of \{ } 3W / 2b \tau_{\text{max}}, \sqrt(3WL / b \sigma_{\text{max}}) \} \] ......(9)

The yield strengths (\( \sigma_{\text{max}} \)) used are 561 MPa for a cast cobalt-chromium alloy\(^{10}\), and 825 MPa for a cast titanium alloy (Ti-6Al-4V)\(^9\).

**RESULTS**

When a cast cobalt-chromium alloy is used

\[ \sigma = 561 \text{ MPa} \]

\[ \tau = 280.5 \text{ MPa} \] (using the Maximum Shear Stress Criterion, a conservative measure, the allowed shear stress is half of yield strength)

Assume the occlusal rest is 2.0 mm (2×10\(^+3\) m) long, with \( W = 200 \text{ N} \), then from (9):

\[ h < \text{max} \{ 3x200 / 2x280.5x10^{+3} b, \sqrt(3x200x2x10^{+3} / 561x10^{+2} b) \} \]

Fig 2a shows the plot of \( b \) (width) against \( h \) (depth or thickness) for a 2.0 mm long cast cobalt-chromium alloy occlusal rest. When the width is 1.0 mm, then the depth or thickness should be at least 1.46 mm. Similarly, when the width is 2.0 mm, then the depth or thickness should be at least 0.85 mm.

When a cast titanium alloy is used

\[ \sigma = 825 \text{ MPa} \]

\[ \tau = 412.5 \text{ MPa} \] (using the Maximum Shear Stress Criterion, a conservative measure, the allowed shear stress is half of yield strength)

Assume the occlusal rest is 2.0 mm (2×10\(^+3\) m) long, with \( W = 200 \text{ N} \), then from (9):

\[ h < \text{max} \{ 3x200 / 2x412.5x10^{+3} b, \sqrt(3x200x2x10^{+3} / 825x10^{+2} b) \} \]

Fig 2b shows the plot of \( b \) (width) against \( h \) (depth or thickness) for a 2.0 mm long cast titanium alloy occlusal rest. When the width is 1.0 mm, then the depth or thickness should be at least 1.21 mm. Similarly, when the width is 2.0 mm, then the depth or thickness should be at least 0.85 mm.

**DISCUSSION**

In this mathematical analysis, the occlusal rest is considered to function as a short rectangular cantilever beam. Under an occlusal force, the bending moment and shear force stresses will increase gradually from the tip of the occlusal rest to its base (Fig 1). The base or junction between the occlusal rest and the minor connector is vulnerable to deformation and fracture\(^2\). Clinically, the occlusal rest is usually spoon-shaped, being narrower and thinner at the tip and wider and thicker at the base. This design complies with the mechanical principles of a cantilever beam. It not only ensures that the occlusal rest can bear the maximum occlusal force without plastic deformation and fracture, but also reduces the amount of tooth tissue removal required.

Although the force selected is simplified as a static vertical occlusal force in this study, forces acting on the occlusal rest will vary widely, depending on functional and other individual circumstances. Also, unlike that for a rectangular beam, the cavosurface angle of the rest seat preparation on an abutment tooth is less than 90 degrees. Therefore, the mathematical analysis predicts only approximate values for the occlusal rest dimensions, and a margin of safety is required, which will then approach the dimensions recommended from clinical experience for cast metal alloys such as cobalt-chromium\(^3,5,7,9,12\).

There are various recommendations concerning the required length for the occlusal rest\(^4,5\). Theoretically, the rest should be as long as possible to allow occlusal forces to be transmitted evenly to the abutment tooth. However, the bending moment increases with increasing length, and the base of the occlusal rest is subjected to the maximum bending force (Fig 1b). A 3-dimensional finite element analysis showed that increasing the rest length from 2.0 to 4.0 mm resulted in some reduction in yield strength (178 to 120\%).\(^6\) Therefore, the 2.0 mm length used in most clinical situations concurs with its use in the mathematical analysis.

The most effective method for maximizing the strength of the occlusal rest is to increase its thickness\(^8\). However, to reduce the possibility of tooth sensitivity and caries risk, the amount of tooth preparation must not exceed the thickness of the enamel\(^9,20,21\). Rigid materials with a high modulus of elasticity should be used to minimize the amount of tooth
preparation needed, while at the same time reducing the risk of deformation and fracture of the occlusal rest. In the mathematical analysis, a cast cobalt-chromium alloy was compared with a cast titanium alloy (Ti-6Al-4V). If the latter material is used for the fabrication of the RPD framework, then approximately 20% less occlusal thickness of tooth substance will be needed to be removed during the rest seat preparation. However, because of titanium's high melting point (~1700 °C), chemical reactivity and low density, special casting equipment and complex laboratory fabrication techniques are required, which are beyond the affordability and capability of many dental laboratories.

The static occlusal force used in the calculations is the average maximum posterior bite force in younger people with an intact dentition, which is approximately three times larger than the average chewing force. The maximum bite force for persons wearing RPDs and for the elderly is usually much lower than for younger persons with an intact dentition. This can explain why wearing RPDs with much smaller occlusal rest dimensions than those generally recommended does not result in fracture of the rests. However, to reduce the risk of deforming or fracturing the occlusal rests, the values derived in this study are suggested as the minimum rest dimensions, especially for younger patients and for those who brux.

CONCLUSIONS

The minimum occlusal rest dimensions for RPDs were predicted by using a mathematical model based on a static occlusal force applied to a short rectangular cantilever beam. For a cast cobalt-chromium alloy, the minimum dimensions for a 2.0 mm long occlusal rest are: when the occlusal width is 1.0 mm the depth or thickness should be at least 1.46 mm; and when the width is 2.0 mm the depth should be at least 1.03 mm. For a cast titanium alloy (Ti-6Al-4V), the corresponding dimensions are: when the occlusal width is 1.0 mm the depth or thickness should be at least 1.03 mm; and when the width is 2.0 mm the depth should be at least 0.85 mm.

ACKNOWLEDGMENT

We would like to acknowledge the matching grant provided by The University of Hong Kong (A/c 20600391) for the support of this project.

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